

Marouane's Method

PRIME NUMBERS FINDING ALGORITHM AND APPROVAL OF THE CONJECTURE OF TWIN PRIMES

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Abstract

This study presents a new prime number finding algorithm, a prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself. A natural number greater than 1 that is not a prime number is called a composite number, in this study we'll build an algorithm that can find all the consecutive prime numbers in a given interval, we'll see testing examples, in the second part of this study we'll prove the twin prime numbers conjecture and give an equation that can find percentage of prime numbers in given interval

Keywords : Prime number, composite number, twin prime number conjecture

Designations:

IC = Impair composite numbers

P = Prime number

P_h = highest prime number in a set

1. INTRODUCTION

An impair positive natural number could be either prime or composite number [1] ex: 5 is impair and is prime but 9 is impair too but composite (not prime) it can be written as $9 = 3 \times 3$.

From here comes the idea, let's find a general relation that shows only the consecutive impairs composite numbers so that we can conclude the impair numbers left, by consequence these impair number must be prime.

2. BUILDING THE ALGORITHM

2.1 Bulding algorithm for an interval [0.100]

Let's write all the impair composite numbers included in the interval [0.100]

{9,15,21,25,27,33,35,39,45,49,51,55,57,63,65,69,75,77,81,85,87,91,93,95,99}

So there are 25 impair composite numbers in the interval [0.100] and the rest except 0 and 1 are all prime number, let's try to find a formula that find all the impair composite numbers in a given interval I so that we can conclude that all the rest in this interval I are prime numbers.

$$\left\{ \begin{array}{l} 3x3 = 9 \\ 3x5 = 15 \\ 3x7 = 21 \\ 3x9 = 27 \\ 3x11 = 33 \\ 3x13 = 39 \\ 3x15 = 45 \\ 3x17 = 51 \\ 3x19 = 57 \\ 3x21 = 63 \\ 3x23 = 69 \\ 3x25 = 75 \\ 3x27 = 81 \\ 3x29 = 87 \\ 3x31 = 93 \\ 3x33 = 99 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 3x(3 + (2x0)) \\ 3x(3 + (2x1)) \\ 3x(3 + (2x2)) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 3x(3 + (2n)), n = 0, 1, 2, \dots, 17 (n \in \mathbb{N}) \end{array} \right. \quad (1)$$

By the formula (1) we found 16 impair numbers in the interval [0.100], we have to find the 9 impairs numbers left {25,35,49,55,65,77,85,91,95}.

We can see that

$$\left\{ \begin{array}{l} 25 = 5x5 \\ 35 = 5x7 \\ 55 = 5x11 \\ 65 = 5x13 \\ 85 = 5x17 \\ 95 = 5x19 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 25 = 5x(5 + (2x0)) \\ 35 = 5x(5 + (2x1)) \\ 55 = 5x(5 + (2x3)) \\ \vdots \\ \vdots \\ 5x(5 + (2n)), n = 0, 1, \dots, 7 (n \in \mathbb{N})(n \neq (2, 5)) \end{array} \right. \quad (2)$$

And

$$\left\{ \begin{array}{l} 49 = 7x7 \\ 77 = 7x11 \\ 91 = 7x13 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 49 = 7x(7 + (2x0)) \\ 77 = (7x(7 + (2x1))) \\ \vdots \\ 7x(7 + (2n)), n = 0, 2, 3 (n \in \mathbb{N})(n \neq (1)) \end{array} \right. \quad (3)$$

So from the sets (1),(2) and (3) we can write a formula to find all the impair composite numbers IC included in the interval $[0,100]$.

$$IC = \sum_{n=0}^{17} \{3x(3+2n)\} + \sum_{\substack{n=0 \\ n \neq (2,5)}}^7 \{5x(5+2n)\} + \sum_{\substack{n=0 \\ n \neq 1}}^3 \{7x(7+2n)\} \quad (n \in \mathbb{N}^+) \quad (4)$$

To simplify formula (4) we can rewrite it as the sum of sets and remove the similar numbers, so equation (4) can be rewritten as:

$$IC = \sum_{n=0}^{17} \{3x(3+2n)\} + \sum_{n=0}^7 \{5x(5+2n)\} + \sum_{n=0}^3 \{7x(7+2n)\} \quad (n \in \mathbb{N}^+) \quad (5)$$

So we've found a relation that can shows all the impairs composite numbers between $[0.100]$, now find all the consecutive prime numbers in this interval will be so easy because all the impair numbers remaining + number 2 are all prime.

Now the question is how to build a general equation for a given interval $[a,b]$?

To answer this question let's try to find for example all the impair composite numbers in the interval $[0,200]$ which are $\{9,15,21,25,27,33,35,39,45,49,51,55,57,63,65,69,75,77,81,85,87,91,93,95,99,105,111,117,123,129,135,141,147,153,159,165,171,177,183,189,195\}$

By using the steps as in the precedent example we find

$$\left\{ \begin{array}{l} 25 = 5x5 \\ 35 = 5x7 \\ 55 = 5x11 \\ 65 = 5x13 \\ 85 = 5x17 \\ 95 = 5x19 \\ 115 = 5x23 \\ 125 = 5x25 \\ 145 = 5x29 \\ 155 = 5x31 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 25 = 5x(5 + (2x0)) \\ 35 = 5x(5 + (2x1)) \\ 55 = 5x(5 + (2x3)) \\ \vdots \\ 5x(5 + (2n)), n = 0, 1, \dots, 16 (n \in \mathbb{N}) \end{array} \right. \quad (6)$$

$$\left\{ \begin{array}{l} 49 = 7x7 \\ 77 = 7x11 \\ 91 = 7x13 \\ 119 = 7x17 \\ 133 = 7x19 \\ 161 = 7x23 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 49 = 7x(7 + (2x0)) \\ 77 = (7x(7 + (2x1)) \\ \vdots \\ 7x(7 + (2n)), n = 0, 2, \dots, 8 (n \in \mathbb{N})(n \neq (1)) \end{array} \right. \quad (7)$$

$$\left\{ \begin{array}{l} 121 = 11x11 \\ 143 = 11x13 \\ 187 = 11x17 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 121 = 11x(11 + (2x0)) \\ 143 = 11x(11 + (2x1)) \\ \vdots \\ 11x(11 + (2n)), n = 0, 2, 3 (n \in \mathbb{N}) \end{array} \right. \quad (8)$$

$$\{169 = 13x13\} \Rightarrow \left\{ \begin{array}{l} 169 = 13x(13 + (2x0)) \\ \vdots \\ 13x(13 + (2n)), n = 0 \end{array} \right. \quad (9)$$

So from equation (11) the prime numbers are all the impair numbers that aren't included in the result of the equation including number 2 , we can also remark that the sets in equation (11) $\{3x(3+2n)\}$, $\{5x(5+2n)\}$, $\{7x(7+2n)\}$, $\{11x(11+2n)\}$, $\{13x(13+2n)\}$ are all function of prime numbers (3,5,7,11,13),so each sets could be written as $Px(P+2n)$ with (P is a prime number).So let's write a general equation for a given interval $[a,b]$.

Let's first calculate the maximum n for each sets in function of a,b and P :

From example 2 we saw that:

$$\{169 = 13x13 \text{ with } 0 < 13x(13+(2n)) < 200 \quad (12)$$

Then (12) could be rewritten in terms of a and b as :

$$a < 13x(13+(2n)) < b \quad (13)$$

By rearranging (13) for n we get :

$$\frac{a-121}{22} < n < \frac{b-121}{22} \quad (14)$$

In the example 2 from equation (11) the maximum prime number in the sets is 13 , so let's write equation (14) in term of P :

13 is the maximum prime number is equation (11) so let's write $P = P_h$ (highest prime number in a set):

$$\frac{a-121}{22} < n < \frac{b-121}{22} \Rightarrow \frac{a-11^2}{2 \times 11} < n < \frac{b-11^2}{2 \times 11} \Rightarrow \frac{a-P^2}{2P} < n < \frac{b-P^2}{2P} \Rightarrow \frac{a-P_h^2}{2P_h} < n < \frac{b-P_h^2}{2P_h} \quad (15)$$

2.2 Theorem:

Be a given interval $I = [a,b]$ and P_i a consecutive series of prime numbers and let P_h be highest prime in the interval $[a,b]$

that satisfies $P_h < \sqrt{b}$, with IC is Impair composite numbers and $\frac{a-P_h^2}{2P_h} < n < \frac{b-P_h^2}{2P_h}$ with ($n \in \mathbb{N}^+$), the new prime numbers

finding equation is defined by:

$$IC = \sum_{n=\frac{a-P_h^2}{2P_h}}^{n=\frac{b-P_h^2}{2P_h}} \{P_i(P_i+2n)\} \quad (n \in \mathbb{N}^+) \quad (16)$$

3. TESTING EXAMPLE

Let's find prime numbers in the interval $I = [200,225]$:

Here we have $a=200$ and $b=225$

Let's find first P_h in this interval I :

$$P_h^2 < b < P_{h+1}^2 \Rightarrow P_h < \sqrt{b} < P_{h+1} \Rightarrow P_h < 15 < P_{h+1}$$

So the highest prime number inferior to 15 is 13 , then $P_h = 13$.

Then all primes from 3 to 13 are: $P_1 = 3$; $P_2 = 5$; $P_3 = 7$; $P_4 = 11$; $P_5 = P_h = 13$

Now that we have calculated P_i let's calculate n for each set

$$\left\{ \begin{array}{l} n_{1\min} = \frac{a - P_1^2}{2P_1} = \frac{200 - 9}{6} = 32 \\ n_{1\max} = \frac{b - P_1^2}{2P_1} = \frac{225 - 9}{6} = 36 \end{array} \right. \left\{ \begin{array}{l} n_{2\min} = \frac{a - P_2^2}{2P_2} = \frac{200 - 25}{10} = 17.5 > 17 \\ n_{2\max} = \frac{b - P_2^2}{2P_2} = \frac{225 - 25}{10} = 20 \end{array} \right. \left\{ \begin{array}{l} n_{3\min} = \frac{a - P_3^2}{2P_3} = \frac{200 - 49}{14} \approx 10.79 > 10 \\ n_{3\max} = \frac{b - P_3^2}{2P_3} = \frac{225 - 49}{14} \approx 12.57 < 13 \end{array} \right.$$

$$\left\{ \begin{array}{l} n_{4\min} = \frac{a - P_4^2}{2P_4} = \frac{200 - 121}{22} \approx 3.59 > 3 \\ n_{4\max} = \frac{b - P_4^2}{2P_4} = \frac{225 - 121}{22} \approx 4.73 < 5 \end{array} \right. \left\{ \begin{array}{l} n_{5\min} = \frac{a - P_5^2}{2P_5} = \frac{200 - 169}{26} \approx 1.19 > 2 \\ n_{5\max} = \frac{b - P_5^2}{2P_5} = \frac{225 - 169}{26} \approx 2.15 < 3 \end{array} \right.$$

So applying the theorem (16) we get the sets :

$$IC = \sum_{n=32}^{36} \{3x(3+2n)\} + \sum_{n=17}^{20} \{5x(5+2n)\} + \sum_{n=10}^{13} \{7x(7+2n)\} + \sum_{n=3}^{5} \{11x(11+2n)\} + \sum_{n=2}^{3} \{13x(13+2n)\}$$

$$IC = \{201, 207, 213, 219, 225\}_1 + \{195, 205, 215, 225\}_2 + \{189, 203, 217, 231\}_3 + \{187, 209, 231\}_4 + \{221, 247\}_5$$

By rearranging in one set and removing the similar numbers and the numbers that are out of the interval $I = [200, 225]$ we get :

$$IC = \{201, 203, 205, 207, 209, 213, 215, 217, 219, 221, 225\} \quad (17)$$

So we have found all the IC (impair composite) numbers included in the interval $I = [200, 225]$, the consecutive missing impair numbers in set (17) are 211 and 223 wich are the **PRIME NUMBERS** we are looking for.

4. CONCLUSION 1

From this new theorem and from the testing example , we conclude that we have found a strong equation that can find all the impair composite and all the prime numbers for a given interval $[a,b]$, we saw in the testing example above that knowing only the prime 13 could help us finding a sequence of primes that are superior at the scare of the highest prime in this interval $P_h^2 < P_n$, hence , this new method could help us finding exactly all the consecutive primes and we've found correlation between primes and impair composite numbers.

5. TWIN PRIME NUMBERS CONJECTURE

A twin prime is a prime number that has a prime gap of two, in other words, differs from another prime number by two, for example the twin prime pair (41, 43). Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair. Twin primes appear despite the general tendency of gaps between adjacent primes to become larger as the numbers themselves get larger due to the prime number theorem (the "average gap" between primes less than n is $\log(n)$) [2].

The twin prime conjecture says that there is an infinite number of such twin pairs. Some attribute the conjecture to the Greek mathematician Euclid of Alexandria, which would make it one of the oldest open problems in mathematics.

To prove this conjecture let's make a graphic interpretation of the theorem (16)

$$IC = \sum_{n=\frac{a-P_i^2}{2P_i}}^{\frac{b-P_i^2}{2P_i}} \{P_i(P_i + 2n)\} \quad (n \in \mathbb{N}^+)$$

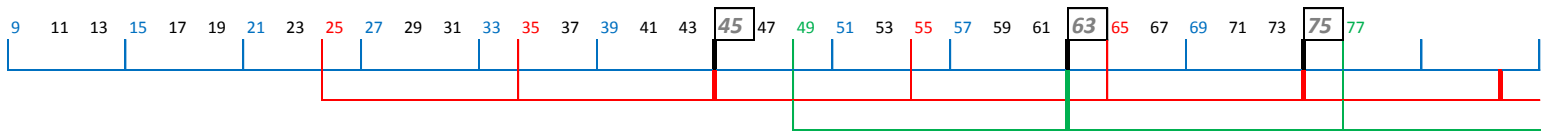


Figure 1

Explanation of the graphic interpretation:

as we follow the theorem (16) to make a graphic interpretation the impair composite numbers in blue represent the sets $(P_3^2 + 2P_3n_3; P_3 = 3; n_3 = 0, 1, 2..)$ while the impair composite numbers in red represent the sets $(P_5^2 + 2P_5n_5; P_5 = 5; n_5 = 0, 1, 2..)$ and the impair composite numbers in green color represent the sets $(P_7^2 + 2P_7n_7; P_7 = 7; n_7 = 0, 1, 2..)$.

From the Figure 1 we see that in the particular case whenever the vertical lines “representing $P_i^2 + 2P_in$ ” are superposed $P_i^2 + 2P_in = P_j^2 + 2P_jn = IC$ with P_i and P_j are primes and $(P_i \neq P_j; n_i \neq n_j)$, we get **twin prime numbers around** this impair composite number IC , for example we can see from the figure 1 when $P_3^2 + 2P_3n_3 = P_7^2 + 2P_7n_7 = 63$, we get twin prime numbers around this $IC = 63$, the twin primes are (63,61). The same thing for $IC = 45$ and $IC = 75$.

Suggestion 1

From here we get the KEY to prove the twin prime numbers conjecture, if there is infinity of impair composite numbers IC that satisfies $(P_3^2 + 2P_3n_3 = P_5^2 + 2P_5n_5 = P_7^2 + 2P_7n_7 = \dots = P_h^2 + 2P_hn_h = IC)$ with $(P_h$ represents the highest prime number in an interval $I = [a, b]$ with $P_h < \sqrt{b} < P_{h+1})$ then we have an infinity of twin prime numbers.

Prove of suggestion 1

So let’s prove the suggestion 1 :

We’ll proceed by recurrence prove, let’s suppose that this impair composite number IC exist for all \mathbb{N} then:

$$\begin{cases} P_3^2 + 2P_3n_3 = IC \\ P_5^2 + 2P_5n_5 = IC \\ P_7^2 + 2P_7n_7 = IC \\ P_{11}^2 + 2P_{11}n_{11} = IC \\ \vdots \\ P_i^2 + 2P_in_i = IC \end{cases} \Rightarrow \begin{cases} P_3^2 + 2P_3n_3 = P_5^2 + 2P_5n_5 \\ P_5^2 + 2P_5n_5 = P_7^2 + 2P_7n_7 \\ P_7^2 + 2P_7n_7 = P_{11}^2 + 2P_{11}n_{11} \\ \vdots \\ P_{i-1}^2 + 2P_{i-1}n_{i-1} = P_i^2 + 2P_in_i \end{cases}$$

We have a system of k equations with k variables, we can then write n_3 in function of $(n_5, n_7, n_{11}, \dots, n_i)$, this system of k equations has a solution then we always have superposition when :

$$n_i = \frac{P_3^2 + 2P_3n_3 - P_i^2}{2P_i} = \frac{P_5^2 + 2P_5n_5 - P_i^2}{2P_i} = \dots = \frac{P_{i-1}^2 + 2P_{i-1}n_{i-1} - P_i^2}{2P_i} \quad (n_1, n_2, \dots, n_i) \in \mathbb{N}$$

How to calculate n_i ?

Let's try to find relation when the vertical lines of ($P_3 = 3; P_5 = 5$) are superposed , hence

$$P_3^2 + 2P_3n_3 = P_5^2 + 2P_5n_5 = IC .$$

Numerical application :

$$9 + 6n_3 = 25 + 10n_5 \Rightarrow n_5 = \frac{6n_3 - 16}{10} \quad (18)$$

When $n_3 = 0$ we get $n_5 = \frac{-16}{10}$ this doesn't make sense because n_5 must satisfy $(n_5) \in \mathbb{N}$ and when $n_3 = 0$, $P_3^2 + 2P_3n_3 = 9$

In this impair composite number 9 we don't have vertical superposition lines according to the figure , se let's write

$$n_5 = \frac{6n_3 - 16}{10} \text{ differently :}$$

$P_3 = 3$ and $P_5 = 5$ have vertical superposition lines for $IC = 45, 75, 105, \dots$

So for

$$P_3^2 + 2P_3n_3 = 45 + 30m \quad (m) \in \mathbb{N}$$

Then

$$P_3^2 + 2P_3n_3 = (9 \times 5) + 30m$$

Hence

$$P_3^2 + 2P_3n_3 = P_3^2 P_5 + 30m$$

So by solving for n_3 we get

$$n_3 = \frac{P_3}{2} (P_5 - 1) + P_5 \cdot m \text{ with } (P_3 < P_5) \text{ and } (m) \in \mathbb{N} \quad (19)$$

And by replacing the values ($P_3 = 3; P_5 = 5$) in n_3 in equation (18) we get:

$$n_3 = 6 + 5 \cdot m \text{ and } n_5 = \frac{20 + 30 \cdot m}{10} \quad (m) \in \mathbb{N} \quad (20)$$

Equation (20) make sense indeed when $m=0$ we get $n_5 = 2$ and $n_3 = 6$

So if we replace respectively $n_5 = 2$ and $n_3 = 6$ in $P_5^2 + 2P_5n_5$ and $P_3^2 + 2P_3n_3$ we get the same $IC = 45$.

By using the equation (19) we can get the same IC for any (P_i, P_j) , for example if we want to know where ($P_5 = 5$) and ($P_{11} = 11$) have the same IC we will apply the equation (19)

$$\text{Equation (19) could be written in general as } n_i = \frac{P_i}{2} (P_j - 1) + P_j \cdot m \text{ with } (P_i < P_j) \text{ and } (m) \in \mathbb{N} \quad (20)$$

So by applying equation (20) we get $n_5 = \frac{P_5}{2} (P_{11} - 1) + P_{11} \cdot m$

Numerical application :

$$n_5 = 25 + 11 \cdot m . \quad (21)$$

For the same IC We have $P_5^2 + 2P_5n_5 = P_{11}^2 + 2P_{11}n_{11} \Rightarrow 25 + 10 \cdot n_5 = 121 + 22 \cdot n_{11}$

Then

$$n_{11} = \frac{10 \cdot n_5 - 96}{22} \quad (22)$$

By replacing equation (21) in (22) we get

$$n_{11} = \frac{110 \cdot m + 154}{22} \quad (m) \in \mathbb{N} .$$

So for example when $m=0$ we get $n_5 = 25$ and $n_{11} = 7$, so we get $P_5^2 + 2P_5n_5 = P_{11}^2 + 2P_{11}n_{11} = 275$

So in this IC=275 the vertical as shown in figure 2 are superposed

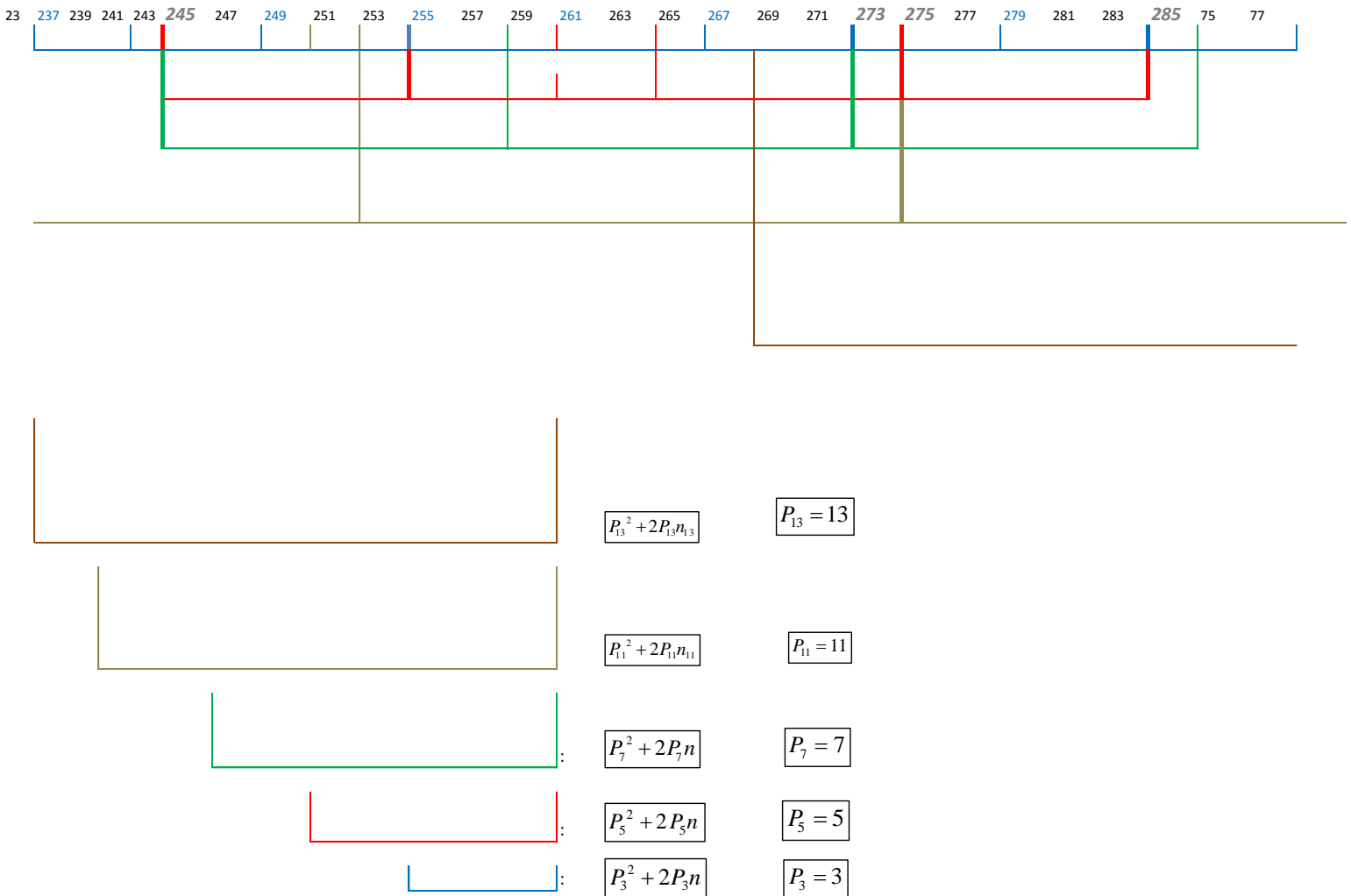


Figure2

In this particular example we see that we don't have twin prime around this IC = 275 because we don't have intersection between lines generated by $P_3 = 3$ and $P_5 = 5$, generally we will get quadruple around an impair composite when all lines are superposed.

Let's make a graphic interpretation of the equation
$$n_i = \frac{P_3^2 + 2P_3n_3 - P_i^2}{2P_i} = \frac{P_5^2 + 2P_5n_5 - P_i^2}{2P_i} = \dots = \frac{P_{i-1}^2 + 2P_{i-1}n_{i-1} - P_i^2}{2P_i}$$

that we saw above and which represent intersection of all sets in given interval and From equation (19) and (20) we see that for different impair composite numbers IC we will get intersections of the form $n_i = a.n_j + b$ which represents an equation of straight line with where a and b are constants are n_j is the slope.

In this example we will take $P_i = P_{17} = 17$ and then $P_{i-1} = P_{13} = 13$, so we will plot straight lines that satisfy :

$$n_{17} = \frac{P_3^2 + 2P_3n_3 - P_{17}^2}{2P_{17}} = \frac{P_5^2 + 2P_5n_5 - P_{17}^2}{2P_{17}} = \frac{P_7^2 + 2P_7n_7 - P_{17}^2}{2P_{17}} = \frac{P_{11}^2 + 2P_{11}n_{11} - P_{17}^2}{2P_{17}} = \frac{P_{13}^2 + 2P_{13}n_{13} - P_{17}^2}{2P_{17}}$$

Then we will replace $P_3, P_5, P_7, P_{11}, P_{13}, P_{17}$ by their respective value 3,5,7,11,13,17, we plot the 5 straight lines in the same graph figure3 :

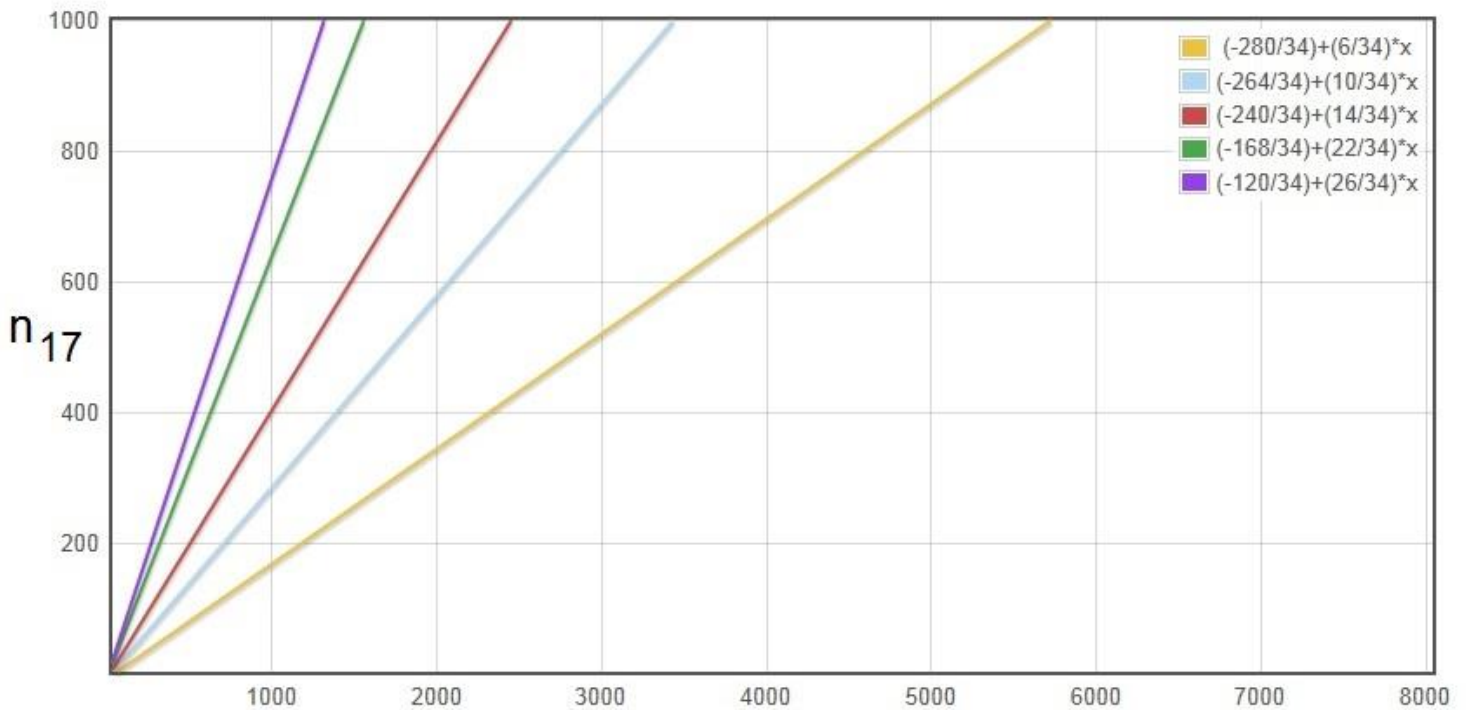


Figure3

The y axis represents values of n_{17} , then we can see that for a same value of n_{17} we'll get different values of $n_3, n_5, n_7, n_{11}, n_{13}$ Which means graphically that intersections that give the same IC (impair composite numbers) for highest values that tend to ∞ are infinite because we'll have infinite straight lines ,this prove that twin, triple and quadruple prime numbers are infinite, we will see below a mathematical approval that twin, triple and quadruple prime numbers are infinite.

6. MATHEMATICAL APPROVAL AND SIGNIFICATION OF INVERSE PRIME NUMBER NUMBERS:

6.1 DEFINITIONS

From figure 1 we can see that vertical lines for $P_3 = 3$ has repetition in each 3 impair numbers beginning from $P_3^2 = 9$ to 15 we have 2/4 points that are for $P_3^2 + 2P_3n_3 = 9 + 6n_3$ and from 9 to 21 we have 3/7 points , from 9 to 27 we have 4/10, then we

have series of the form: $\frac{2}{4} + \frac{3}{7} + \frac{4}{10} + \dots + \frac{2+k}{4+3k}$ with $k = 0, 1, 2, \dots, \mathbb{N}$.

Actually this series show the percentage that $P_3 = 3$ occupy in all impair numbers then:

$$\lim_{k \rightarrow \infty} \frac{2+k}{4+3k} = \frac{1}{3} \approx 0.33$$

This give us a signification that $P_3 = 3$ is repeated at a percentage of 33%, if we repeat the same process for $P_5 = 5$ we have repetition in each 5 points beginning from $P_5^2 = 25$ to 35 we have 2/6 and from 25 to 45 we have 3/11 .. then we have series of

the form $\frac{2}{6} + \frac{3}{11} + \frac{4}{16} + \dots + \frac{2+k}{6+5k}$ $k = 0, 1, 2, \dots, \mathbb{N}$ so the percentage that $P_5 = 5$ occupy in all impair numbers is:

$$\lim_{k \rightarrow \infty} \frac{2+k}{6+5k} = \frac{1}{5} = 0.20,$$

so $P_5 = 5$ is repeated at a percentage of 20%.

We by repeating the same process for $P_7 = 7, P_{11} = 11, P_{13} = 13, \dots, P_i = i$ we find respectively ($1/7, 1/11, 1/13, \dots, 1/i$)

So if we want to calculate the total percentage that occupy all Prime numbers in all impair numbers , we have to calculate

$\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots + \frac{1}{P_i} = \sum_{i=0}^{\infty} \frac{1}{P_i}$ with (P is prime number and $P_0 = 3$), this is the sum of the reciprocals of all prime numbers wich is diverging in $+\infty$ [3].

This means 2 things for us , the numbers of the prime are infinite and another thing , each parameter in the sum represents the percentage of a prime number in all impair numbers . Normally the percentage must be $<100\%$,so something is missing in this sum ? The answer is yes because as we saw above their is intersections that we have to eliminate in the sum, and according to the theorem (16) we have to eliminate also the gap from $P_3^2 = 9$ to the beginning of the next set please refer to figure1 , for example : $P_5^2 + 2P_5n_5$ begins from 25 after jumping 8 impair numbers when $n_5 = 0$ and $P_7^2 + 2P_7n_7$ begins from 49 after jumping 20 impair numbers when $n_7 = 0$, so we have to eliminate respectively the gap $\frac{25-9}{2} = 8$, and $\frac{49-9}{2} = 20$ and so on,

then the general relation for the gap is $g = \frac{P_i^2 - 9}{2}$ and to eliminate the gap as term of percentage in given interval $I = [a, b]$ is :

$\%g = 1 - \frac{b-g}{b}$, for example in interval $I = [0, 600]$ the gap for P_5 is $g(P_5) = \frac{25-9}{2} = 8$, then the percentage og the gap in

this interval with $b=600$ is $\%g = 1 - \frac{600-8}{600} \approx 1.33\%$, that means practically that all impair numbers that can be divided by

$P_5 = 5$ are about 20% as we saw above but we've to eliminate this gap then $20\% - 1.33\% \approx 18.67\%$.

Now that we know how to calculate the percentage of the gap let's try to know how to calculate the percentage of intersection. We've seen earlier how to calculate intersections between 2 series of prime numbers, so for example intersection between lines generated by $P_3 = 3$ and $P_5 = 5$ is given by :

$$n_3 = 45 + 30m_3$$

Graphically we'll have intersection at every 15 impair numbers then the percentage of intersection of lines generated by $P_3 = 3$ and $P_5 = 5$ is given by:

$$\frac{1}{\frac{30}{2}} = \frac{1}{15} = \frac{1}{P_3 P_5} \approx 6.67\% .$$

Between $P_3 = 3$ and $P_7 = 7$ is $(63 + 42.m_3^*)$ with $(m_3^* \neq m_3)$ then the percentage is:

$$\frac{1}{\frac{42}{2}} = \frac{1}{21} = \frac{1}{P_3 P_7} \approx 4.76\% .$$

6.2 NUMBER OF INTERSECTIONS.

We saw previously in paragraph 6.1 that percentage of intersections between vertical lines generated by 2 prime numbers P_i and P_j is given by $\frac{1}{P_i P_j}$, we will soon to calculate the sum of percentages of all intersections , then we must know the number

of intersections(NI), we will give an example then we will generalize.

Let's find the percentage of intersections of lines generated by these 4 prime numbers ($P_0 = 3, P_1 = 5, P_2 = 7, P_3 = 11$), then :

$$IL = \left[\begin{array}{l} (P_0^2 + 2P_0n) \cap (P_1^2 + 2P_1n) \\ (P_0^2 + 2P_0n) \cap (P_2^2 + 2P_2n) \\ (P_0^2 + 2P_0n) \cap (P_3^2 + 2P_3n) \end{array} \right] + \left[\begin{array}{l} (P_1^2 + 2P_1n) \cap (P_2^2 + 2P_2n) \\ (P_1^2 + 2P_1n) \cap (P_3^2 + 2P_3n) \end{array} \right] + \left[(P_2^2 + 2P_2n) \cap (P_3^2 + 2P_3n) \right] \quad (23)$$

$$\begin{aligned}
IL &= \left(\frac{1}{P_0P_1} + \frac{1}{P_0P_2} + \frac{1}{P_0P_3}\right) + \left(\frac{1}{P_1P_2} + \frac{1}{P_1P_3}\right) + \left(\frac{1}{P_2P_3}\right) \\
IL &= \left[\frac{1}{P_0}\left(\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}\right)\right] + \left[\frac{1}{P_1}\left(\frac{1}{P_2} + \frac{1}{P_3}\right)\right] + \left[\frac{1}{P_2}\left(\frac{1}{P_3}\right)\right] \\
IL &= \left(\frac{1}{P_0} \sum_{i=0}^2 \frac{1}{P_{i+1}}\right) + \left(\frac{1}{P_1} \sum_{i=0}^1 \frac{1}{P_{i+2}}\right) + \left(\frac{1}{P_2} \sum_{i=0}^0 \frac{1}{P_{i+3}}\right) \tag{24}
\end{aligned}$$

Equation (23) shows The number of intersections of vertical lines generated by the 4 prime numbers (3,5,7,11) , this number which we'll call it NI equal to 6 in this example , then:

$$NI = (4-1) + (4-2) + (4-3) = \sum_{n=1}^{4-1} 4-n$$

Then in general the number of intersections for lines equal to the number of prime numbers in a given x impair numbers is given by:

$$NI = \sum_{n=1}^{\pi(x)-1} \pi(x) - n \quad \text{With } (\pi(x) \text{ is the number of prime numbers}) \tag{25}$$

Obviously when $\pi(x)$ tends to $+\infty$ NI tends to $+\infty$, this means that the number of intersection of vertical lines tends to $+\infty$ when the number of prime numbers tends to $+\infty$, this another **proof of the TWIN PRIME CONJECTURE.**

in general equation (24) for given $\pi(x)$ is defined by:

$$IL = \sum_{i=0}^{\pi(x)-1} \left(\frac{1}{P_i} \sum_{i=0}^{\pi(x)-2} \frac{1}{P_{i+1}}\right) \tag{21}$$

So now that we can calculate the percentage of the gap and the percentage of intersections of vertical line generated by primes numbers and as we know to calculate the percentage of impair composite numbers generated by prime numbers as shown in the theorem (16) , we can calculate percentage of all impairs composite numbers and therefore the percentage of prime numbers , then the formula will be %IC (percentage of impair composite numbers = sum of percentages of impairs composite numbers generated by prime number – percentage of intersections of all superposed vertical lines – percentages of the gap of all prime numbers then mathematically the formula is :

$$\%IC = \sum_{i=0}^{\infty} \frac{1}{P_i} - \sum_{i=0}^{\infty} \left(\frac{1}{P_i} \sum_{i=0}^{\infty} \frac{1}{P_{i+1}}\right) - \sum \%g \tag{22}$$

The sum (22) is converging I calculated it in the interval $I = [9, 400]$ then primes participating are (3,5,7,11,17,19) because

$$P_{19}^2 < 400 < P_{23}^2 \text{ and I get}$$

$\%IC \approx 62\%$, this means that in the interval $I = [9, 400]$ 62% of impair numbers are not prime therefore, the percentage of prime numbers are $1-0.62= 38\%$ then the percentage of prime numbers in this interval is 38%

6.1 APPROVAL OF THE TWIN PRIME CONJECTURE

We saw above that we gave a prove of infinity of intersections by system of equation and then we gave another prove graphically, so let's prove it mathematically.

From equation (21) we saw that intersection of vertical lines generated by primes numbers is given by :

$$IL = \sum_{i=0}^{\infty} \frac{1}{P_i} \sum_j \frac{1}{P_{j+1}}$$

This equation represent the percentage of the sum of the intersections of vertical lines generated by primes numbers , in this particular case we'll have quadruple prime number , so if this case is infinite then the twin prime numbers are infinite,

$$\sum_{i=0}^{\infty} \frac{1}{P_i} \sum_j \frac{1}{P_{j+1}} = \lim_{\substack{i \rightarrow \infty \\ j \rightarrow \infty}} \frac{1}{P_i} \cdot \frac{1}{P_{j+1}} = \lim_{i \rightarrow \infty} \frac{1}{P_i} \cdot \lim_{j \rightarrow \infty} \frac{1}{P_{j+1}} = +\infty \quad \text{Because we know that } \lim_{i \rightarrow \infty} \frac{1}{P_i} = +\infty.$$

Then the number of intersections of vertical lines generated by prime number is infinite, from here we conclude that TWIN PRIME NUMBERS and QUADRUPLE PRIME NUMBERS are infinite.

5. REFERENCES:

[1] Wikipedia website http://en.wikipedia.org/wiki/Prime_number

[2] Wikipedia website http://en.wikipedia.org/wiki/Twin_prime

[3] Wikipedia website http://en.wikipedia.org/wiki/Divergence_of_the_sum_of_the_reciprocals_of_the_primes